

# The Evolution of Latent Health over the Life Course

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## **Abstract**

We propose a new method to estimate rich dynamic models of health that exploits longitudinal observations of multiple health measures. Our two-step approach combines factor analysis with simulation methods, two techniques first developed in different contexts. In the first step, we use factor analytic methods to estimate age-specific static measurement models that determine how latent health is related to observed health measures. This step also recovers the unconditional nonparametric distribution of latent health at each age. During the second step, we use the method of simulated moments to estimate a stochastic dynamic model of latent health. Specifically, we simulate the dynamic health process and use the previously estimated measurement model to derive an implied set of moments that we can compare with moments in the data. We demonstrate the method by estimating health processes using data from the Health and Retirement Study. Our findings

show the importance of using multiple health measures to estimate dynamic models of health. Our estimates based on multiple health measures display significantly less persistence in health than do standard estimates obtained using single measures of health.

## 1 Introduction

Elderly individuals' economic decisions depend on their health and how it changes with age. As physical health deteriorates, the capacity to work declines, leading some to retire from the workforce.<sup>1</sup> Health also affects how much the elderly consume and save, both because health influences preferences directly and because the elderly save for future, uncertain medical expenditures. To understand the economic choices the elderly make, one needs to understand how health evolves stochastically with age. In recent years, a literature has developed that estimates dynamic stochastic models of latent health using health indicators available in recent panel surveys.<sup>2</sup> In particular, this literature uses dynamic panel methods based on observing a single health measure, most often the self-rated health status (SRHS) measure, in multiple periods. SRHS is very persistent across ages, which leads this literature to find that health differences across individuals are very

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<sup>1</sup>There is a large literature on the role of health in retirement. In a recent contribution, Bound, Stinebrickner, and Waidmann (2010) find that health plays a central role in retirement behavior. They report that at age 62, those in poor health exit the labor force at five times the rate of those in good health.

<sup>2</sup>Examples are Contoyannis, Jones and Rice (2004), Halliday (2008, 2010), and Heiss, Börsch-Supan, Hurd, and Wise (2009).

persistent. Clearly, however, any method based on a single health measure depends critically on the validity of that measure. The measure must contain little noise, it should be unbiased, and it should capture most aspects of health. Although SRHS is widely accepted among researchers (including us) as very informative, it is unlikely that any single health measure can satisfy all these requirements. It is unlikely that the narrow empirical basis that single health measures such as SRHS provide can adequately represent the dynamics of health. However, our understanding of the economic choices of the elderly will remain incomplete as long as we cannot represent the dynamics of health well. We therefore propose to enlarge the empirical basis for estimating the dynamics of health to include a broad and diverse set of health measures. The methodological challenge that we meet in this paper is to develop and estimate dynamic models of health that are both sufficiently tractable to be useful in standard structural models of economic behavior and based on multiple health measures that provide a broad, encompassing description of physical health.<sup>3</sup>

Increasingly, individual micro-data sets, often with a panel dimension, collect additional measures of health beyond SRHS. For example, the Health and Retirement Study (HRS) contains other self-reported health measures including an Index of the Ability to perform Daily Activities (IADL) and an index of the ability to perform task that require general physical strength

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<sup>3</sup>We do not consider mental health in this paper, although it would be straightforward (even if numerically expensive) to extend the analysis to include mental health measures.

(the “large muscle index”). The HRS also collects multiple clinical measures such as a grip strength measure, a timed walk measure, and a measure of lung functioning. All these measures contain information that can be exploited to describe health more broadly than if one simply relies on SRHS. When we estimate our dynamic model, we use information from four different measures of health available in the HRS panel, including two clinical measures. As such, our estimates have a much richer empirical basis than existing approaches.

We conceive of health as a single latent index that evolves stochastically over time. This latent health index maps onto measures of health observed in the data. The challenge is to estimate both how health and health measures are jointly distributed and how the health index itself evolves stochastically with age. We split this problem into two parts. We first use a factor analytic approach to estimate how health and health measures are jointly distributed within age, then we estimate the stochastic process of health itself.

Our two-step approach based on both clinical and self-reported measures of health has a number of methodological advantages for estimating the joint distribution of health and health measures in the cross-section. First, we estimate how health and health measures are jointly distributed conditional on age before we estimate how health evolves with age. Consequently, our estimates of the joint distribution of health and health measures conditional on age are robust to misspecifying the dynamic process of health. Second, we allow the relation between physical health and respondents’ perceptions of

their health to change with age. For example, a level of physical health that might be reported as “fair” at age 50 could be perceived to be “excellent” at age 85. To anchor how health is distributed at different ages, we use the clinical measures. Third, we can identify a nonparametric distribution of latent health because we exploit continuous measures of health. Unlike previous work, we do not impose normality but instead allow the distribution of health to exhibit skew and thick tails. We find these features of non-normality to be important in the cross-section.

We consider three alternative dynamic models of health. The simplest model treats the health index as a first-order autoregressive process.<sup>4</sup> The second specification augments the basic model with an endogenous mortality equation. That is, we allow survival to depend on health and we estimate this dependence in the model. The third model allows for measurement-specific random effects. This model is motivated by the observation that the auto-correlations of measurements across age are much larger than those observed within age across measurements. To account for this fact, we allow for individual random effects specific to each measurement.

Mirroring the standard findings in the literature, the basic specification

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<sup>4</sup>We also considered a model that allows for an asymmetry in the regression to the mean in the dynamic process. That is, we allowed the autoregressive component to be of different strength depending on whether the individual is healthier or sicker than the average individual in the population. This model was motivated by the intuitive notion that recovery from sickness differs qualitatively from health differences between individuals who are relatively healthy. When we estimated that model, we did not find strong evidence for asymmetries in the regression to the mean and therefore do not report estimates from that model here.

displays a surprising degree of persistence in health. The autoregressive parameter is close to 1 and the variance in the health innovations is very low. Only among men over 75 is there evidence for regression to the mean. Once we allow for endogenous mortality, we find more rapid declines in health among the oldest population, which are partially offset by dynamic selection from the bottom of the health distribution. Allowing for endogenous mortality, however, has little effect on how much persistence we find in health. We still observe regression to the mean only among the oldest men and we find little variance in health innovations. However, once we allow for measurement-specific random effects, we find evidence for regression to the mean for both genders and among most age groups. We also find that the variance of health innovations increases by up to an order of magnitude for all ages and both genders. Models with measurement-specific random effects fit the data substantially better. Our findings support the need for multiple measurements of health in estimating the dynamics of health in order to distinguish persistence in individual health measurements from persistence in health overall.

Although we find much less persistence in the random effects model, we caution against overstating this finding. Regardless of the model we estimate, we find large health differences in the population at age 50 that tend to persist over the remainder of the life-cycle.

Our work is related to multiple literatures, which we cannot adequately review in the space available here. One important literature has made use of

the multiple health measures in the HRS to address concerns that justification bias in self-rated health might give rise to the large observed relations between self-rated health and retirement behavior. Here, the work by John Bound and co-authors (in particular, Bound (1991) and Bound, et al. (2010)) deserves particular mention.<sup>5</sup> These studies rely on exogeneity assumptions related to a subset of variables that is perceived to be less likely to be subject to justification bias to implement either structural or instrumental variable methods to estimate effects of health on retirement that are free of justification bias. These studies do not focus on the dynamics of health or the relation between latent health and the observed health measures. More closely related to our work is the recent work on the dynamics of health exemplified by Contoyannis, et al. (2004), Halliday (2008, 2010), and Heiss, Börsch-Supan, Hurd, and Wise (2009). These studies attempt to estimate latent variable models of health similar to our basic specification.<sup>6</sup> The fundamental difference between these studies and ours is that they rely on single indicators of health such as SRHS (Contoyannis et al. (2004), Halliday (2010) and Heiss, et al. (2009)) or an inability to work measure (Heiss et al. (2009)), sometimes combined with a mortality equation. They are therefore based on substantially less information than our approach, are sensitive to the specific measure used to represent health, and fail to fit the empirical joint distribution of the

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<sup>5</sup>See also Blau and Gilleskie (2001)

<sup>6</sup>Related literature (Adda, Banks, and von Gaudecker (2009), Adams, Hurd, McFadden, Merrill, and Ribeiro (2003), Borsch-Supan, Heiss, and Hurd (2003), Contoyannis, Jones, and Rice (2004), Halliday (2008)) studies how health and income evolve jointly using dynamic panel data methods.

multiple health measures within and across age. By construction, they cannot distinguish between persistence of latent health and persistence of the unique health measures they use. Some of their conclusions on persistence have to be modified once one exploits a broader set of health variables.

Overall, we hope that our paper contributes to what is, in the words of Halliday (2010), still “very much a fledgling field” by expanding the empirical basis for dynamic latent health models to include multiple health measures. We propose a new estimation method and provide estimates of dynamic models of health that successfully describe how health measures at different ages are jointly distributed. At the same time, these dynamic models still maintain a tractable, single-index specification for health.

We begin our exploration in Section 2 by describing the data. In Section 3, we present the static factor model that relates health measures to the underlying health index. We show estimates of this model in Section 4. Section 5 presents the dynamic specifications, and Section 6 discusses the method we use to estimate the dynamic models. In Section 7, we discuss estimates of the dynamic models, and we conclude in Section 8.

## **2 Measures of Health in the Health and Retirement Study (HRS)**

The Health and Retirement Study (HRS) is uniquely suited for analyzing the dynamics of health among the elderly. It contains detailed health measures



for a panel of individuals who have been followed for up to 16 years. In this section, we describe the sample and health measures we use in our study. In particular, we present reduced form results that motivate why we rely on multiple health measures to study the dynamics of health. These results also motivate how we specify the dynamics of health in our preferred model.

We argue that the traditional focus on SRHS misses much of the useful information on health contained in the HRS. Two observations support this argument; there is a lot of systematic variation in the additional health measures that SRHS cannot explain and this variation correlates with important outcomes such as subsequent mortality, labor force participation, and weekly earnings. We then use principle component analysis (PCA) to show that it is possible to capture much of the information in the available health measures using a single-index model of health augmented by a set of factors specific to the various health measures. In the cross-section, these measurement-specific components are akin to uncorrelated measurement error. However, dynamically the measurement-specific components correlate across age. Overall, our descriptive analysis of these measures therefore leads us to formulate single-index factor models of health in the cross-section that, in our preferred specification, contain individual measurement-specific random effects.

## **2.1 The Data**

The HRS is a large representative longitudinal survey with both clinical and self-reported measures of health. The clinical measures are continuous, which

will allow us to estimate nonparametric distributions of health. The self-reported measures are categorical variables. Every two years since 1992, the HRS has surveyed nearly 20,000 respondents representing the US population aged 50 and older. In order to maintain a representative sample of the population aged 50+, new birth cohorts are enrolled every 6 years. We base our study on the nine surveys conducted between 1992 and 2008.<sup>7</sup>

Our data is based on the RAND HRS data files, the HRS Tracker files and the physical measure files for the 2004-2008 waves. The RAND files (version H) are a user-friendly version of the HRS made available by the RAND corporation. We obtain from it the self-reported health variables, age, and height. We add three clinical health measures (peak expiratory air flow, hand grip strength and timed walking speed data) from the 2004-2008 physical measure files. The HRS Tracker files (version 2.0, January 2008) provide vital statistics based on the National Death Index (NDI).<sup>8</sup>

Table 1 documents summary statistics for our sample. We use 158,595 observation years obtained from 29,723 individuals. 5,769 individuals report data for all 9 surveys. The average age in the sample is 67 and 44% of respondents are male. The NDI covers the period from 1992 to 2004 and applies to 25,803 individuals. 26% of these individuals had died by 2004 at

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<sup>7</sup>We use data from annual surveys between 1992 and 1996 and the biannual surveys between 1996 and 2008.

<sup>8</sup>The mortality data in the tracker files is based on finder files submitted to NCHS in 1995, 2000, 2002, and 2004. Based on the information in the tracker files, we can determine the vital status and the year and month of death up to 2004.

an average age just over 78 years old. A final control used in this analysis is a measure of individual height, which we demeaned by gender. The standard deviation in individual height is just over seven centimeters.

The HRS collects a multitude of self-reported health measures and we chose two from these: an index of large muscle strength (LMI) and SRHS. SRHS is reported on a five point scale from poor to excellent. The LMI is derived from variables indicating difficulties in four tasks: sitting for two hours; getting up from a chair; stooping, kneeling, or crouching; and pushing or pulling a large object.<sup>9</sup> In addition to the self-reported health measures, we use three clinical health measures collected since 2004. Hand grip strength measures general muscle strength and whether respondents suffer from arthritis and other conditions in the hand. Hand grip strength has been shown to be related to general physical and medical status and predicts mortality. The grip strength variable is roughly log normally distributed and we therefore use the log of hand grip strength in our analysis. The measure of lung function, peak expiratory air flow, is strongly indicative of obstructive lung disease. Declines in peak expiratory air flow have been shown to be related to mortality, as well as to cognitive and physical decline. Finally, the timed walk, which has only been collected from individuals over 65, has been shown to be a highly reliable measure of functional capacity that predicts many major health outcomes.

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<sup>9</sup>SRHS is available for all waves in the study, whereas the LMI is unavailable for wave 1 and for a subset of the sample in wave 2.

As documented in Table 1, we find that respondents on average report difficulties in 1.3 (out of four) tasks that make up the LMI and report themselves to be in good health. The LMI and SRHS indicate that the population was slightly less healthy after 2004, which is due to the population being somewhat older during these later years. As is also evident from Table 1, we have many fewer observations for the clinical health measures than the self-reported measures. This is because these measures have been collected only since 2004 and they have been administered only to random subsets of the sample. For example, the expiratory air flow and log grip strength measures were collected from approximately 3,000 respondents in 2004 and 7,000 each in 2006 and 2008. Unfortunately, only a few individuals were administered the clinical measurements in two adjacent years, which makes them less useful to estimate dynamic models. Finally, the timed walk is only available for individuals over 65.<sup>10</sup> For these reasons, the self-reported measures will be more influential in our estimation results.

## 2.2 The Case for Going Beyond SRHS

Just because measures are available does not mean that we should use them. It is plausible that a summary measure such as SRHS captures most of the relevant information on health. In that case, using additional health variables makes the analysis unnecessarily complex. We therefore will show that SRHS

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<sup>10</sup>We use the timed walk to estimate the static measurement model, but we do not currently use it to estimate the dynamic model.

fails to fully capture the systematic variation in the health measures included in our analysis and that these additional measures provide additional information for explaining outcomes such as mortality, work participation, and earnings.

Table 2 shows two correlation matrices for males in our sample (results for females are similar and available on request). Panel A shows the correlations for all five health measures in the data after residualizing on gender, height of the individual, and a full set of age indicators. All these correlations are highly statistically significant. The largest is the correlation between the two self-reported measures - SRHS and the LMI. Generally, a clinical measure will be more highly correlated with another clinical measure than with a self-reported measure. Similarly, the two self-reported measures are more highly correlated with each other than with any of the clinical measures.

Panel B then shows that the additional health measures are correlated with each other even after residualizing on SRHS. Although the correlations do decline, they all remain positive and highly significant. Clearly, these measures are systematically related even after removing the component explained by SRHS, and they contain additional information about the health of the individual.

In Table 3 we relate these health measures to meaningful outcomes such as mortality, whether an individual works, and weekly wages among those working. In columns 1 and 2, we estimate a hazard model relating mortality

to SRHS and LMI and test whether the LMI is significant.<sup>11</sup> We then consider regression models of work participation and weekly earnings. We report results with and without the clinical measures, because the latter are available for only a small subset of the sample.

The SRHS measure clearly contains a lot of information for both the mortality and the work decision. For instance, mortality risk for those who report "very good" or "excellent" health is only about one-fifth of the mortality risk for those reporting "poor" health. Similarly, work participation rates and wages are substantially higher among those reporting better health. The magnitude of the relation between SRHS and these outcomes is large enough to explain a significant proportion of the data.

The other health measures, however, also explain relevant proportions of the data. For instance, the partial R-squared associated with the additional health measures is about one-half to two-thirds as large as the R-squared associated with SRHS. And, these additional measures are highly statistically significant explanatory variables for mortality and work, and to a lesser degree wages. Clearly, the full vector of health measures explains much larger fractions of the variation in outcomes than does SRHS on its own.

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<sup>11</sup>We can not relate mortality to the physical measures, because these are only collected in the HRS after 2004 and the mortality data are only available up to 2004.

### 2.3 Health as a Scalar?

Our approach summarizes how health measures are correlated in the cross-section using a single health factor. Using the correlations of health measures of individuals of the same age, we estimate a factor model. This produces estimates of how the scalar "health" is distributed and how the health measures are distributed conditional on the health factor. Together these two components explain the joint distribution of health measures within age.

One might question whether a single-index model can capture the correlation-structure across health measures observed in the cross-section. To answer this question we perform a PCA on the correlation matrix. We perform this analysis separately by 5-year age group and gender and present the results for the males 65-69 years old in Table 4.<sup>12</sup>

In PCA, the eigenvalues determine the proportion of the total variation in the health measures that is explained by the associated orthogonal factors (principal components).<sup>13</sup> The principal components are ordered by the sizes of their eigenvalues - that is by the amount of variation each explains. From Table 4, we see that the first principal component explains 38% of the total variation in the data. Examining the remaining eigenvalues, we find that these account for a much smaller proportion of the variation and that they account for roughly equivalent amounts (between 11% and 19%). This

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<sup>12</sup>The results from this age group are similar to those obtained for females and for those from other age groups and are available from the authors on request.

<sup>13</sup>Our results are based on the correlation matrix and are thus scale invariant and assign equal importance to the variation in each measure.

pattern is consistent with a data structure in which a single factor (health) explains the cross-correlations across the measures and the remaining variation is captured by orthogonal, measurement-specific factors.

Concerning the factor loadings, we find that all health measures load on the first principal component with roughly equal coefficients. We interpret this as evidence that there is indeed a systematic scalar source of variation that drives much of the variation in health measures and can reasonably be interpreted as health. There is an interesting pattern among the loadings on the second principal comp within self-reported health measures that we already commented on when examining Table 2: some respondents tend to self-report worse health than seems indicated by their clinical measures. This results in relatively high agreement between the two self-reported health measures as well as between the three clinical measures, whereas there is less agreement between self-reported and clinical measures. A more complete analysis would require a second factor to capture subjective attitudes toward health that are not confirmed in the physical measures. We find it difficult to interpret the remaining principal components.

Overall, the patterns in the eigenvalues and eigenvectors from PCA suggest that a model with a single health factor can account for much of how this set of health measures covary across individuals. Given our preference for parsimony, we refrain from using higher dimensional factor models and instead concentrate our efforts on a single-factor model.



## 2.4 Correlations in Health Measures across Age

The same health measures from different ages are highly correlated. Table 5 shows the pairwise correlations between our health measures within age as well as the correlations of the health measures with those obtained two years prior. SRHS has an autocorrelation of 0.66, indicating a significant amount of persistence. The autocorrelation for LMI is 0.62, and those for the physical measures are similarly high. At the same time, and as we noted above, the correlations within age across measures are much smaller - the largest being 0.44 for the SRHS and LMI measures. The correlations across age and measures are still smaller. These features of the correlations suggest a significant systematic component for each measure that persists over time and is specific to that measure. Our preferred dynamic specification will therefore include measurement-specific random effects.

In the next two sections, we describe the static model and its estimation before we turn to the dynamic model.

## 3 The Static Measurement Model

For each 5-year age group and gender we represent the joint distribution of self-reported and clinical health measures using a single-factor model. We refer to this scalar factor as health and denote it by  $h_a$ , where  $a$  stands for age. The factor model specifies how health is distributed and how the health measures relate to individual height and age, to the factor  $h_a$ , and to

measurement-specific stochastic error. The parameters to be estimated for each 5-year age group are the parameters entering the distribution of  $h_a$  and the parameters of each measurement equation.<sup>14</sup>

To develop this structure, we need additional notation. Let  $Y_a$  stand for an  $m_1$ -vector of categorical health indicators  $y_{j,a}$  and  $X_a$  for an  $m_2$ -vector of continuous health measures  $x_{j,a}$ . The total number of categorical and continuous measurements is  $m = m_1 + m_2$ . Each categorical variable  $y_{j,a} \in Y_a$  is assumed to reflect an underlying latent index  $\tilde{y}_{j,a}$ . The categorical variables are ordered and have  $K_j$  segments defined by cutoffs  $c_{j,g}^k$ . Thus, each  $y_{j,a} \in Y_a$  is linked to its latent counterpart  $\tilde{y}_{j,a} \in \tilde{Y}_a$  :

$$y_{j,a} = \sum_{k=1}^{K_j} 1(\tilde{y}_{j,a} \geq c_{j,g}^{k-1}) \quad (1)$$

where  $1(\cdot)$  is an indicator function taking the value 1 if the condition in parentheses is true and 0 otherwise. The latent indices  $\tilde{y}_{j,a}$  and the continuous measurement variables  $x_{j,a}$  are collected in a vector  $Z_a$ .

We then relate the measurements  $Z_a$  to the health factor  $h_a$  as well as  $r$  additional controls  $Q_a$ , and a vector of measurement-specific errors  $\epsilon_a$ . The controls  $Q_a$  include age (within age group) and height. Height proxies for the direct effect of physical capacity that is unrelated to health on the clinical

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<sup>14</sup>The youngest age group covers the ages 50–54. The oldest age group covers a longer interval and includes all respondents aged 85–100. We drop respondents over 100.

measures such as log grip strength, expiratory airflow, and walking speed.

$$Z_a = \begin{pmatrix} X_a \\ \tilde{Y}_a \end{pmatrix} = \alpha_g + \Lambda'_g h_a + \Theta'_g Q_a + \epsilon_a \quad (2)$$

Equations (1) and (2) define the measurement model. Here,  $\alpha_g$  denotes an  $m$ -vector of intercepts,  $\Lambda_g$  an  $m$ -vector of factor loadings,  $\Theta_g$  an  $m$ -by- $r$  matrix of regression coefficients, and  $\epsilon_a$  an  $m$ -vector of independently distributed measurement errors.<sup>15</sup> The parameters to be estimated include the factor loadings, the intercepts for the continuous measures, the cut-offs for the categorical measures, the regression coefficients, the variances of  $\epsilon_a$  for the continuous measures, and the parameters that govern the distribution of the health factor  $h_a$ .<sup>16</sup> All of the parameters in this measurement model are subscripted by  $g$  because we estimate the measurement model separately for each gender and 5-year age group. Thus, we rely on only a cross-section of health measures for age group  $g$  to identify the "static" parameters  $(\alpha_g, \Lambda_g, Q_g, Var_g(\epsilon_a))$  and the distribution  $F_g(h_a)$ .

Besides the distribution  $F_g(\cdot)$  of the latent health variable  $h_a$ , we need to identify the parameters  $(\alpha_g, \Lambda_g, Q_g)$  and the vector of the variances of  $\epsilon_a$ .<sup>17</sup> Standard factor analytic results imply that, assuming that we have three or more measurement variables, we can identify these parameters up to two

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<sup>15</sup>The assumption of independence is stronger than the uncorrelatedness assumption typical in factor analysis. It is required to obtain identification of the distribution of  $h_a$  without parameteric assumptions.

<sup>16</sup>For the ordered categorical measures, the usual normalizations imply that the intercept is 0 and the variance of the measurement error is 1.

<sup>17</sup>We provide a more complete identification argument in Appendix 1.

normalizations, one for the scale and one for the location of the distribution of the latent health variable. We normalize the intercept and the factor loading of the log grip strength variable.

Normalizing the intercept to 0 and the coefficient on the  $h_a$  to 1 in the measurement equation for log grip strength implies that health is measured in units of log grip strength. That is, a one-unit increase in health implies an increase in expected log grip strength by one unit. Further, the average of health conditional on age equals the average log grip strength for that age. To the extent that the log grip strength is an objective, interpretable measure of health, we can use it to compare the level of health across ages. However, grip strength itself is difficult to interpret in terms of outcomes that individuals care about. Therefore, we will use the estimates of the mortality model obtained in Section 6 to renormalize the health factor and describe the distribution of health in units of predicted mortality.

A second set of normalizations accounts for the fact that the self-reported measures are ordinal, categorical variables without scale or location. We normalize the intercepts and the error variances of the measurement equations dealing with categorical variables to 0 and 1 respectively.

As we show in Appendix 1, our approach has the major advantage that we can nonparametrically identify how health is distributed. This result relies on having more than one continuous measures of health, such as the log grip strength or the expiratory air flow measure. In practice, we need to impose some parametric functional form. We assume that the latent health variable

is distributed as a mixture of two normal random variables with different means and variances:

$$F_g(h_a) = p_g * N(\mu_{1,g} + \beta_{1,g}a, \sigma_{1,g}^2) + (1 - p_g) N(\mu_{2,g} + \beta_{2,g}a, \sigma_{2,g}^2) \quad (3)$$

The means of the component normal distributions depend linearly on age. The distribution of health conditional on age therefore contains seven parameters. These are the mixture probability  $p_g$  as well as the three parameters that govern each of the mixture distributions. Mixtures of normal distributions are very flexible and can accommodate skewness, thick tails, and bimodalities that cannot exist in a single normally distributed variable.

## 4 Estimating the Static Measurement Model

To estimate the static model, we pool all observations in the HRS from the same age group and gender. We define eight age groups ranging from 50-54 to 85+. For each combination of gender and age, we estimate the specification described in Section 3 using Mplus. We have searched intensively over the parameter space to find global maxima. Mplus provides us with asymptotic standard errors, which we report in this section. When we combine the static and the dynamic models, we bootstrap the estimation procedure to obtain consistent standard errors for all the model parameters.

Depending on the age group, the measurement model contains either 25

or 29 parameters for a total of 440 parameters. These are too many to discuss in this paper and we therefore present estimates for males and females from 3 age groups (50-54, 65-69, 80-84) only. The estimation results from the other age groups are comparable and available on request. Table 6 reports point estimates for the parameters in the measurement model (2).

Table 6

Across all age groups, the estimated parameters are qualitatively similar even if they sometimes differ in magnitude. For instance, the factor loading for SRHS is typically about one-third larger than that for LMI and about 4-7 times the size of the (normalized) loading on the log grip strength variable. The loadings on the air flow and the walking speed measures are by contrast 1-2 times the size of the loading on the log grip strength variable. Similarly, the error variances are quite similar across age groups. Generally, the loadings and the error variances are precisely estimated.

We consider next the regression coefficients on age for the self-reported health measures. These age coefficients are based only on how health responses vary with age within age group relative to changes in the actual health factor. The health factor itself also depends on age, and we present estimates of these associations below. The changes in the health factor with age (within age group)<sup>18</sup> are driven by the average decline in clinical mea-

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<sup>18</sup>The changes in the health factor across age groups (not across age within age group) reflect the average decline in the normalized health measure, that is, the log grip strength

asures. The regression coefficients on age for the self-reported health measures represent average changes in self-reported health with age that are not also reflected in changes in the clinical measures. Positive coefficients on age such as those observed in the data signify that individuals report better health as they age than is indicated by the physical capabilities reflected in the clinical measures. This pattern suggests that respondent self-reports of health are at least partially informed by comparisons of their own health with the health of others of the same age.

The regression coefficients on height are positive for the clinical measures, but less so for the self-reported measures. For the LMI, we find either a zero coefficient or a negative relation with height. For SRHS, we find a positive relation with height. However, comparing the regression coefficient on height with the factor loading for SRHS, we find that the relation between height and SRHS is much weaker than that between the health factor and SRHS. For the clinical measures, we find that height has a strong impact, especially on the grip strength variable, suggesting that taller individuals are generally stronger. By controlling for height in the measurement model, we remove any long-run correlations between height and health. It is not clear that this is the appropriate specification, because height itself may be correlated with health. Nevertheless, to the extent that height is fixed (we measure height as the maximum height reported in the panel and it is thus fixed across age), we believe that the dynamic specifications estimated below will be unaffected.

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measure.

In Table 7, we show how the measurement model fits the data using moments obtained from 50-54 year old females. Clearly, the model fits the joint distribution of LMI and SRHS well. We have only the log grip strength variable and the expiratory air flow variable for the years 2004 and 2008, and only for a subset of observations during those years, so there are far fewer observations with both variables ( $N=842$  vs.  $N=11,254$ ). This implies that the parameter estimates of the static model will primarily be driven by the self-reported health measures. Nevertheless, we also capture most of the joint variation in the clinical measures. We interpret these results as evidence that the static measurement model in fact captures the variation in health measures within an age group very well.

Table 7

Figure 1 shows how average observed and fitted health measures as well as the health factor vary with age.

Figure 1

The model fits the declines in the averages of the various health measures well. By construction, the decline in the health factor mirrors the decline in log grip strength closely.

We consider now how the health factor is distributed. For this, recall that the measurement model is normalized against the log grip strength measure.



Consequently, a unit change in the health factor corresponds to a unit change in the expected log grip strength.<sup>19</sup> Thus, the distribution in the health factor is meaningful to the extent that one can interpret the distribution of expected grip strength. We acknowledge that grip strength itself is not a particularly interesting outcome. We will therefore revisit the distribution of health after we have estimated the mortality model and will then express health in terms of predicted mortality rates.

Nevertheless, it is of interest to examine the distribution of health even when normalized against the log grip strength variable because it informs us of asymmetries in the distribution of health and whether it is necessary to account for them. The distribution of health is governed by seven parameters: the variance, the intercept, and age-coefficient for the mean of the two mixture distributions in addition to the weight placed on each of the component distributions. Table 8 presents these estimates for the same three age groups and both genders. The parameters themselves are difficult to interpret, so we also display the means, standard deviation, and skew. We plot the estimated densities in Figure 2.

Table 8

Figure 2

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<sup>19</sup>The grip strength variable itself has been standardized to have a mean of 0 and a standard deviation of 1 across the entire population.

Mean health according to this health factor declines with age and is lower for females than for males. This gender difference reflects that males are generally stronger physically than females. We believe that these gender differences should not be interpreted as reflecting real health differences - they underscore the need to separately estimate the health models for males and females.

Health is not normally distributed, but characterized by left skew. This skew is stronger for the younger population, but we find significant deviations from normality even at older ages. Further, we find that health among males is skewed more heavily to the left. Our finding that the distribution of health is non-normal stands in contrast to the normality assumptions standard in the literature on the dynamics of health (e.g. Halliday (2010) and Heiss et al. (2009)).

In summary, we find that our health model fits the data in the age-conditional cross-sections well and that the underlying distribution of health is not normal. In addition, the measurement model (2) delivers an estimate of the conditional distribution of measures of health that we require to estimate the dynamic model.

## 5 The Dynamics of Health

We estimate three dynamic models of health. In all three the health scalar  $h_a$  follows a first-order autoregressive process with drift:

$$h_{a+1} = \mu_g + \rho_g h_a + v_a \quad (4)$$

The parameters that govern this annual process are the constant  $\mu_g$ , the autoregressive parameter  $\rho_g$ , and the variance of the innovation  $\sigma_{v,g}^2$ . The innovation  $v_a$  captures idiosyncratic shocks to individuals' health and is assumed to be normally distributed. The parameters are indexed by  $g$ , which denotes the 5-year age group of the individual in year  $a$ . This allows the dynamic process governing health to change with age.<sup>20</sup>

To start the random process (4), we require a distribution  $F(h_0)$  that describes how health is distributed in the initial period. We use our estimate of how health is distributed among 50-54 year old males and females, respectively. This distribution is described in Table 7 and Figure 2 above.

Our basic dynamic specification consists of this initial distribution together with the dynamic equation (4). By combining this basic specification with the measurement model (2) we obtain an implied joint distribution of health measures across time. This joint distribution depends on the dynamic parameters  $(\mu_g, \rho_g, \sigma_{v,g}^2)$  and on the parameters of the measurement model.

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<sup>20</sup>In principle, we could allow the parameters to vary for each age of the individuals, but limited sample size forces us to group individuals into 5-year age groups.

Our second model augments the basic specification with a mortality equation:

$$\Pr(s_a = 1) = \Phi(\alpha_{0,g} + \alpha_{1,g}h_a) \quad (5)$$

where  $s_a$  is an indicator for whether an individual survives to age  $a+1$  conditional on being alive at age  $a$ , and  $\Phi(\cdot)$  denotes the standard normal cumulative distribution function. Again, we allow the parameters to vary by age group. Estimating the dynamic model now means estimating  $(\mu_g, \rho_g, \sigma_{v,g}^2)$  and  $(\alpha_{0,g}, \alpha_{1,g})$  for all age groups.

Our third model is motivated by the correlation patterns documented in Table 5. The autocorrelations within health measures across age are much higher than the correlations observed between health measures within or across age. We assume that the measurement error  $\varepsilon_a^m$  pertaining to the measure  $m$  at age  $a$  is composed of an age-constant random effect  $\chi^m$  and an age-specific measurement error  $\omega_a^m$ .<sup>21</sup>

$$\varepsilon_a^m = \chi^m + \omega_a^m \quad (6)$$

We assume that  $\chi^m$  is normally distributed with variance  $\sigma_{\chi,m}^2$  and is uncorrelated with (i) the health factor  $h_a$ , (ii) all other random effects  $\chi^{k \neq m}$  and (iii) all age-specific measurement errors  $\omega_a^k$ . Furthermore, we assume that the measurement error  $\omega_a^m$  is normally distributed with a variance  $\sigma_{\omega,m,g}^2$  that is allowed to differ across age groups  $g$ . Estimating the model with random

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<sup>21</sup>These heterogeneous  $\varepsilon_{a,i}^m$  are collected in the random vector  $\epsilon_a$  in (2).

effects requires estimating an additional four age-invariant parameters  $\sigma_{\chi,m}^2$  that govern the variance of the random effects.<sup>22</sup>

We will now describe the general method we use to estimate our three dynamic models.

## 6 Estimating the Dynamic Model

In this section we describe how we estimate the parameters of the dynamic process. Our general approach combines a separately estimated comprehensive measurement model with the method of simulated moments.

### 6.1 A Simulated Method of Moments Approach to Estimating the Dynamic Latent Health Process

We propose a simulation-based algorithm that minimizes the distance between moments obtained from simulated health measurement data and moments of the empirical distribution of measures. Let  $\tilde{Z}$  denote a simulated panel data-set containing measures of health for individuals at different ages. On the basis of  $\tilde{Z}$  we can compute simulated moments  $\tilde{M}(\theta)$  that depend on the parameter vector  $\theta$  governing the dynamic model of health. We construct the same moments from the observed data and denote these  $M$ . Our estimator then chooses  $\hat{\theta}$  to minimize the distance

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<sup>22</sup>The variances of the age-specific measurement error terms do not need to be estimated as they are determined by the variances of the random effects and the error terms estimated for static model (2) above.

$D(M - \widetilde{M}(\theta)) = (M - \widetilde{M}(\theta))' W (M - \widetilde{M}(\theta))$ , where  $W$  is an appropriately chosen weighting matrix.

The contents of  $\theta$  depend on the model that is estimated. For the most basic model, the parameter vector  $\theta = \{\mu_g, \rho_g, \sigma_{v,g}^2\}_{g=0}^G$  comprises 24 parameters. When we correct for mortality,  $\theta$  also contains the mortality parameters for each age group:  $\theta = \{\mu_g, \rho_g, \sigma_{v,g}^2, \alpha_{0,g}, \alpha_{1,g}\}_{g=0}^G$ . In that case  $\theta$  has dimension 40. Finally, the random effects model depends on a parameter vector  $\theta$  that consists of 44 parameters:  $\left\{ \{\mu_g, \rho_g, \sigma_{v,g}^2, \alpha_{0,g}, \alpha_{1,g}\}_{g=0}^G, \{\sigma_{\chi,m}^2\}_{m=1}^M \right\}$ .

At the core of the estimator is an algorithm that lets us construct  $\widetilde{M}(\theta)$ . Before we explain this algorithm, note that we have separately estimated how the latent health variable maps into the health measures  $Z_a$  (as shown in Sections 3 and 4). We can thus treat this mapping as known. Furthermore, note that the static measurement model estimated at age 50 provides an estimate of the initial distribution  $F(h_0)$  of health for the starting age.

The algorithm to construct the simulated moments of measurements  $\widetilde{M}(\theta_1)$  implied by any parameter realization  $\theta_1$  consists of the following steps:

- Step 1: Generate draws of initial health  $\widetilde{h}_0$  for a large simulated sample of individuals by drawing from the estimated distribution  $F_0(\cdot)$  of the latent health variable at the initial age  $a = 0$ .
- Step 2: If the dynamic model contains random effects, use  $\theta_1$  to generate  $\{\chi_m\}_{m=1}^M$  for each individual in the simulated sample.
- Step 3: Use the dynamic model with the parameters  $\theta_1$  to draw  $\widetilde{h}_{a+1}|\widetilde{h}_a$  for

each individual. This generates a simulated panel of health histories.

Step 4: Use the implied survival probability  $\Phi\left(\alpha_{0,g} + \alpha_{1,g}\tilde{h}_a\right)$  to simulate the mortality process and generate a sample of survivors.

Step 5: For each age in the panel, use the estimates from the static measurement model together with the sample of random effects  $\chi_m$  to draw  $\tilde{z}_a|\tilde{h}_a, \chi_m$  for the sample of survivors.

This leaves us with a panel of measurements  $\{\tilde{z}_a\}_{a=0}^A$  from which we can generate  $\tilde{M}(\theta_1)$  and for which we can generate the distance  $D(M - \tilde{M}(\theta_1))$ . It should be clear that this algorithm can be implemented for more complex dynamic processes than we have shown here. The main constraints that prevent us from estimating richer dynamic models are computational.

## 6.2 Implementing the Simulated Moments Algorithm

In order to implement the above algorithm we must choose the appropriate set of moments  $M(\theta)$ . By construction, the measurement model matches the cross-sectional distribution of the health measures in a given age group. The moments that are available to estimate the dynamic models are therefore moments from the joint distribution of health measures across time. The HRS imposes a further restriction in that observations are spaced two years apart. We therefore match the following moments for each age group:

1. For each continuous clinical measure we match

- (a) average change in the measure from age  $a$  to  $a + 2$ . (2 moments)
  - (b) variances in the clinical measures in  $a + 2$  (2 moments).
  - (c) The covariances of the measures (both within and across measures) between  $a$  and  $a+2$  (4 moments).<sup>23</sup>
2. For each categorical self-reported measure, we match the entire intertemporal transition matrix. Both LMI and SRHS have five support points, so there are 25 transition probabilities. The transition probabilities within each row of the transition matrix must sum to 1, implying five restrictions on the transition matrix. This means that each of the ordinal categorical variables contributes 20 moments and we have an additional 40 moments by going across categories (80 moments).
  3. We also use the expected value of each continuous variable in  $a + 2$  conditional on the support of each categorical variable in  $a$  (20 moments).
  4. To identify the survival process, we also match
    - (a) mean of the clinical measures in  $a$ , conditional on dying before period  $a + 2$  (two moments)
    - (b) marginal distribution for each of the self-reported measures, also conditional on dying before period  $a + 2$  (eight moments)
    - (c) unconditional mortality rate (one moment)

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<sup>23</sup>The true covariance of log grip strength in period  $a$  and peak expiratory air flow in period  $a+2$  is the same as the covariance of these variables in the opposite periods, but because these moments can be different in the data, we match both.



For each age group, we thus have a total of 119 moments to estimate  $\theta$ . The estimated parameters are asymptotically consistent regardless of the weighting matrix chosen. However, in a finite sample, the choice of the weighting matrix will result in different estimates. We use a grid search to find good starting values and thus need to compare the minimized criterion function across different starting values to ensure that we find the global minimum. This precludes using the conventional two-step optimal weighting scheme, because in this scheme the minimized criterion depends on the weight matrix, which is itself a function of the initial parameters. Instead, we use the inverse of the variance matrix of the observed moments, which places extra weight on precisely estimated moments.

We bootstrap the standard errors at the individual level using 100 replications. We estimate both the measurement model and the dynamic model within each bootstrap replication and thus obtain standard errors that account for the estimation error in the measurement model.

## **7 Estimates of the Dynamic Model**

### **7.1 The Fit of the Model**

In Tables 9, 10, and 11, we present the parameters of the three dynamic models of health. We discuss what these parameters mean in more detail below. At this point, we simply consider how well these models fit the dynamic aspects of the data.

We compare the observed and the fitted correlation matrices of the four health measures used to estimate the dynamic models: log grip strength, expiratory air flow, SRHS, and LMI. In Table 12, we show for women between 65 and 69 how the observed and the predicted correlations compare.<sup>24</sup>

As we discussed in Section 2, health measures correlate highly across age within but not across measures, which motivated the random effects model. Unsurprisingly, the basic specification and the mortality-corrected model do not capture these high autocorrelations specific to individual measures. Instead, we find they obtain very similar correlations across age regardless of whether we consider the correlation within the same measure or across different measures. For example, the basic and the mortality model predict that the autocorrelation of the expiratory air flow measure is approximately 0.17. These models find a similar correlation of the expiratory air flow measure with the log grip strength measure across age and a larger correlation with the two self-reported measures. These findings are clearly at odds with the empirical facts. By contrast, the random effects model fits the data quite well. It allows both for the high autocorrelations within health measures and for the lower correlations across health measures and time. The overall dynamic fit of the model is thus substantially improved by including measurement-specific random effects in the error component of the models.

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<sup>24</sup>We get a similar fit for the other groups.

## 7.2 The Units of Health: More on the Distribution of Health

Because of an arbitrary normalization, we measure health in units of log grip strength. Any linear transformation of the health factor is equally capable of capturing the correlation in health measures. Furthermore, once we describe the health dynamics with a factor  $h$  that follows some dynamic model, we are free to take an arbitrary monotone transformation  $g(h)$  to assign meaning to the latent variable health. Rather than measuring health in units of log grip strength, we use our estimate of the mortality model (eq. 5) to measure health in predicted mortality rates. Transforming the health factor  $h$  into mortality units is useful because mortality units are a more familiar outcome and because it is easier to attach value to mortality risk.

The first panel of Table 13 reproduces the health distribution based on the log grip strength normalization, previously shown in Table 7 and Figure 2. The second and third panels display the distribution of health in mortality units. The second panel is based on expected mortality itself and is measured in annual percent mortality risk, whereas the third panel shows the distribution of mortality risk relative to the average risk for each age and gender. In Figure 3, we show the distribution of health as measured in predicted mortality.

Both the average mortality and the variation in predicted mortality risk in the population rise sharply with age. The average mortality risk for females

aged 80-84 is for instance about five times the risk for women aged 50-54, whereas the standard deviation in mortality risk for the older group exceeds that of younger women by a factor of 6. At the same time, the health distribution displays right skew for all ages, so there is a sizeable minority that has dramatically higher risk in all age groups. The skew is somewhat smaller for the older group, but nevertheless the increased dispersion in mortality risk means that there are large differences in predicted mortality in this population at all ages. For example, the 90-10 ratio in predicted mortality is between 1.75 and 2 for all age and gender groups.

As Table 13 and Figure 3 reveal, the health distribution is skewed, leading the average predicted mortality rate to exceed that at the median by only between 5% and 10% depending on the age and gender group. We judge this degree of skew to be relatively small compared to the overall variation in health observed in this data. There is substantial dispersion in health as measured by expected mortality rates within age and gender group.

### **7.3 The Persistence and Volatility of Health**

How persistent are health differences between individuals across age? Can the sick and sickly expect to recover over time or are their health difficulties likely to remain with them? The answers to these questions differ across the three specifications described above. In particular, we find that the two models that do not account for the measurement-specific random effects predict more persistence and significantly less volatility in health than does our preferred

specification that accounts for the persistence specific to each measurement. Our results suggest that dynamic models that do not allow for measurement-specific persistent differences will attribute much of the persistence in health measures observed across individuals to general health, thus overestimating the stability of health differences in the population.

We consider first the estimates of the basic specification (Table 9) as well as the mortality-corrected specification (Table 10).<sup>25</sup> For both models, the parameter estimates display a considerable degree of persistence. The AR-1 parameter is typically close to 1, suggesting that health closely resembles a random walk. For older men do we find AR-1 coefficients that are below 1, but only for the oldest group can we reject a random walk. Overall, these estimates suggest that individuals do not recover from health difficulties.<sup>26</sup> Instead of observing regression to the mean in health, existing health differences in the population seem to persist as individuals age. The volatility estimated for these specifications is likewise small. Annually, the standard deviation in the innovations is typically below one-tenth and sometimes below one-hundredth of the overall standard deviation in health observed in the population (see Table 8). These estimates of the basic specification thus sug-

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<sup>25</sup>In each table, the last column summarizes the average decline in health implied by the parameters. This decline is calculated by taking  $E[h_{a+1}] - E[h_a] = E[\mu + \rho h_a + \varepsilon_{a+1}] - E[h_a] = \mu + (\rho - 1) E[h_a]$  and depends on the two parameters  $(\mu, \rho)$  as well as the observed mean health for each age:  $E[h_a]$ . For the mortality model, this decline is calculated without conditioning on survival - it shows what would happen to average health in the population if individuals were not subject to mortality.

<sup>26</sup>This is true for the frequency that the HRS is collected: every two years. It is possible that there is more significant regression to the mean at much shorter frequencies. The HRS is poorly suited for examining health dynamics at very short frequencies.

gest that health is a very stable process and that health differences between individuals are very persistent across ages.

The estimates of the random effects model in Table 11, however, lead us to revise this conclusion. The auto-regressive parameters for women are uniformly smaller than those reported in Table 10. Among older women and especially men, the point estimate of  $\rho$  suggests a strong tendency for health to regress back to the mean. Health is also much more volatile than previously thought. Almost uniformly, the estimated variances in innovations from the random effects model exceed those obtained in the basic specifications: some of them are much greater, particularly for older individuals.

Overall, there is still a substantial degree of persistence in the population, but compared to the estimates in Tables 9 and 10, the more complete model allows for a significant degree of regression to the mean among men and older women. And, the observed volatility now implies that a nontrivial fraction of the population experiences larger declines in health than the remainder of the population. This is particularly true among the elderly.

## 8 Conclusion

In this paper, we have proposed a new simulation-based method of estimating how health evolves as individuals age. Our approach splits the problem into two parts. The first is a static measurement stage that recovers for each age how latent health (conditional on controls) is distributed and delivers

age-specific estimates of how latent health maps into observed health measurements. The second stage focuses on the dynamics. Using the estimates of the measurement model, we can generate simulated joint distributions of the manifest measurement variables implied by our model of dynamic health and our estimated measurement model. The estimation proceeds by choosing parameters of the dynamic model that minimize differences between cross-age moments in the simulated and observed distributions of measures.

To demonstrate our method, we have estimated a measurement model using five health measures available in the HRS and, using this measurement model, estimated three different dynamic models of health. Our estimates of the static measurement model allow us to consider nonparametric estimates of the distribution of health. Using expected mortality to set the scale of the health distribution, we find that health is non-normally distributed and displays significant left skew, reflecting the observation that a significant fraction in the population registers low values on multiple or all health measures. We also find that the variance in expected mortality increases significantly alongside the average mortality rate as individuals age.

Our simplest dynamic health model displays a very high degree of persistence. According to this model, health evolves as a random walk with very little volatility. When we correct for endogenous mortality selection, health is still very persistence and follows a dynamic process that closely resembles a random walk. However, these two models fail to adequately describe the dynamics of health measures we observed in the data. In particular, these

two models are incapable of reproducing the high degree of autocorrelation in specific health measures we observed in the data. To account for this, we allow for measurement-specific random effects and find that this substantially improves model fit. When we allow for random effects specific to the measures, health displays a greater tendency to regress to the mean and is substantially more volatile across age.

## 9 Appendix: Identification of the Basic Model

In this appendix, we discuss how the static measurement model and the basic dynamic model are identified.<sup>27</sup> The parameters that need to be identified are the dynamic parameters  $(\mu_a, \rho_a, \sigma_{v,a}^2)$ , the initial distribution of health  $F(h_0)$ , and the parameters from the measurement model  $(\alpha_a, \Lambda_a, c_{j,a}^{k-1})$ . We will show that these parameters are identified up to a normalization on the intercept and factor loading for one of the continuous measurement equations, as well as the standard normalizations on variances and intercepts of categorical measurement equations. We assume that we have access to at least two continuous measurement variables and three additional continuous or categorical measurement variables.

First, we appeal to standard factor analytic arguments and assert that with three continuous and categorical measurement variables, we can identify

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<sup>27</sup>To simplify the notation, we omit the direct dependence of the measurement equations on age and on height. Extending the identification to allow for control variables is unproblematic.



the parameter vectors  $(\alpha_a, \lambda_a)$  and the variances of  $\epsilon_a$  up to a normalization of one intercept and one factor loading. We will impose these normalizations on the same measurement equation at all ages.

We did not restrict the distribution of  $F(h_a)$  and we therefore need to show, using Kotlarski's Theorem, that  $F(h_a)$  can be nonparameterically identified using two continuous measurements only. Use the first and second continuous measurement for this identification argument:  $x_{1,a}$  and  $x_{2,a}$ . We have normalized the factor loading and intercept on the first and thus have

$$\begin{aligned} x_{1,a} &= h_a + \varepsilon_{1,a} \\ x_{2,a} &= \alpha_{2,a} + \lambda_{2,a}h_a + \varepsilon_{2,a} \end{aligned}$$

Because  $(\alpha_2, \lambda_2)$  are identified, we can write:

$$\begin{aligned} x_{1,a} &= h_a + \varepsilon_{1,a} \\ \frac{x_{2,a} - \alpha_{2,a}}{\lambda_{2,a}} &= h_a + \frac{\varepsilon_{2,a}}{\lambda_{2,a}} \end{aligned}$$

and can treat the left-hand side of both of these equations as observed. Kotlarski's Theorem implies that if  $\left(h_a, \varepsilon_{1,a}, \frac{\varepsilon_{2,a}}{\lambda_{2,a}}\right)$  are jointly independent and  $E[\varepsilon_{1,a}] = E\left[\frac{\varepsilon_{2,a}}{\lambda_{2,a}}\right] = 0$ , then the marginal distribution of  $h$  can be identified from the joint distribution of  $(x_{1,a}, x_{2,a})$ . Therefore,  $F(h_a)$  and the parameters of the measurement equations are identified.

We have yet to discuss the identification of the parameters of the dynamic

equation (4). For this purpose, we will restrict attention to two adjacent ages  $(a, a + 1)$ . First, note that we can identify the parameters  $\{\mu_a\}$  using the marginal distributions of health  $h_a$  directly:

$$\begin{aligned}\mu_0 &= E[h_0] \\ E[h_{a+1}] &= \mu_a + \rho_a E[h_a]\end{aligned}$$

Now, from equation (4) we get:

$$\begin{aligned}E[h_{a+1}|Z_a] &= \mu_a + \rho_a E[h_a|Z_a] + E[\varepsilon_{a+1}|Z_a] \\ \implies \rho_a &= \frac{E[h_{a+1}|Z_a] - \mu_a}{E[h_a|Z_a]}\end{aligned}$$

$E[h_a|Z_a]$  can be directly obtained using the parameter estimates from the measurement model. However, we lack direct estimates of  $E[h_{a+1}|Z_a]$ .<sup>28</sup> However, we have the following:

$$\begin{aligned}E[Z_{a+1}|Z_a] &= E[\alpha_{a+1} + \Lambda_{a+1}h_{a+1} + \varepsilon_{a+1}|Z_a] \\ &= \alpha_{a+1} + \Lambda_{a+1}E[h_{a+1}|Z_a] \\ \Leftrightarrow E[h_{a+1}|Z_a] &= (\Lambda'_{a+1}\Lambda_{a+1})^{-1}\Lambda'_{a+1}(E[Z_{a+1}|Z_a] - \alpha_{a+1})\end{aligned}$$

where the right-hand side can be obtained using the estimated factor loading matrices and the data. Therefore  $\rho_a$  is identified.

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<sup>28</sup>Note that  $E[h_{a+1}|Z_a] \neq E[E[h_{a+1}|Z_{a+1}]|Z_a]$ .

To identify  $\sigma_a^2$ , consider the following expression:

$$\begin{aligned}
\text{Var}(h_{a+1}|Z_a) &= V(\mu_a + \rho_a h_a + \varepsilon_{a+1}|Z_a) \\
&= \rho_a^2 V(h_a|Z_a) + V(\varepsilon_{a+1}|Z_a) \\
\implies \sigma_a^2 &= \text{Var}(h_{a+1}|Z_a) - \rho_a^2 V(h_a|Z_a)
\end{aligned}$$

Again, we obtain  $V(h_a|Z_a)$  directly from the measurement model and we need to concern ourselves only with finding  $\text{Var}(h_{a+1}|Z_a)$ . For this purpose, we again use the joint distribution of the measurement equations.

$$\begin{aligned}
V(Z_{a+1}|Z_a) &= V(\alpha_{a+1} + \Lambda_{a+1} h_{a+1} + \varepsilon_{a+1}|Z_a) = \Lambda_{a+1} V(h_{a+1}|Z_a) \Lambda'_{a+1} + V(\varepsilon_{a+1}|Z_a) \\
&= \Lambda_{a+1} V(h_{a+1}|Z_a) \Lambda'_{a+1} + V(\varepsilon_{a+1}) \\
\Leftrightarrow V(h_{a+1}|Z_a) &= (\Lambda'_{a+1} \Lambda_{a+1})^{-1} \Lambda'_{a+1} (V(Z_{a+1}|Z_a) - V(\varepsilon_{a+1})) \Lambda_{a+1} (\Lambda'_{a+1} \Lambda_{a+1})^{-1}
\end{aligned}$$

where again the right-hand side is observed or estimable from the static model.

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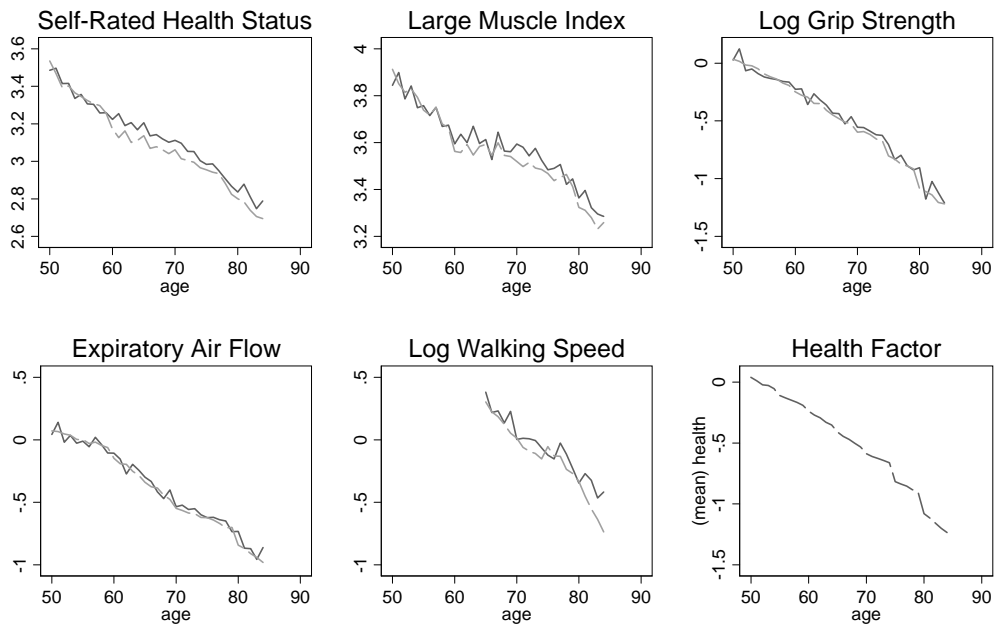
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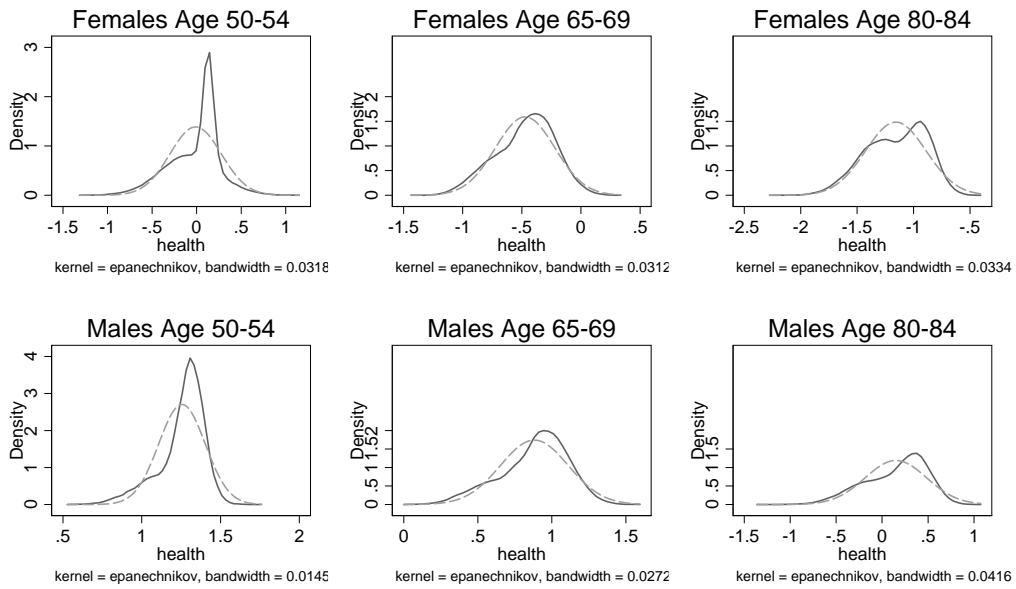
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Figure 1: Mean Female Health Measures by Age



Solid lines depict the observed and broken line the simulated health measures

Figure 2: Estimated Health Factor Densities  
Measured in Expected Log Grip Strength Units



Also shown are normal densities with same mean and variance.

Table 1: Summary Statistics

	<i>Full Sample: 1992-2008</i>		<i>Only 2004-2008</i>	
	<i>Mean</i>	<i>Std Dev</i>	<i>Mean</i>	<i>Std Dev</i>
Self-rated Health Status	3.14	1.16	3.09	1.13
Large Muscle Index	3.74	1.32	3.66	1.33
Exp. Air Flow	na	na	0.00	1.00
Log Grip Strength	na	na	0.00	1.00
Log Walking Speed	na	na	0.00	1.00
Age	66.95	10.61	68.58	10.57
Height	0.00	0.07	0.03	0.07
Fraction: Male	0.44		0.44	
Observations	158,595		53,902	

Reported are summary statistics for the full HRS sample as well as for the 2004 - 2008 years only. The objective health measures are available during the 2004-2008 period and for subsamples of the total sample. We have recoded the Large Muscle Index and the log speed variable so that higher values represent better health. The Large Muscle Index is coded as 5 minus the number of (up to) 4 activities for which a respondent reported difficulties. The Self-rated Health Status takes integer values running from 1 to 5. The objective measures are standardized to mean zero and standard deviation 1 in the full sample. The height of individuals is standardized within gender to have a mean of zero and is measured in meters. The physical measures are available for only about 17,000 respondent years for the grip strength and expiratory air flow measure and about 10,000 for the walking speed measure. The latter variable is only available for respondents aged 65 or more.

Table 2: Standard Dichotomous Health Measures Capture Only Part of Information in Health Measures

Panel A: Health Measures Residualized on Gender, Height, and Age.

	<i>Self-Rated Health Status</i>	<i>Large Muscle Index</i>	<i>Peak Expiratory Air Flow</i>	<i>Log Grip Strength</i>	<i>Log Walking Speed</i>
Self-Rated Health Status	1				
Large Muscle Index	0.44	1			
Peak Expiratory Air Flow	0.28	0.18	1		
Log Grip Strength	0.19	0.19	0.27	1	
Log Walking Speed	0.24	0.21	0.27	0.23	1

Panel B: Health Measures Residualized on SHRS, Gender, Height, and Age.

	<i>Self-Rated Health Status</i>	<i>Large Muscle Index</i>	<i>Peak Expiratory Air Flow</i>	<i>Log Grip Strength</i>	<i>Log Walking Speed</i>
Self-Rated Health Status	na				
Large Muscle Index	na	1			
Peak Expiratory Air Flow	na	0.06	1		
Log Grip Strength	na	0.11	0.23	1	
Log Walking Speed	na	0.12	0.22	0.19	1

Notes: Reported are correlations between the residualized health measures. In Panel A, the health measures have been residualized using a regression of the various health variables on height, gender and a full set of age dummies. In Panel B, the health measures have been residualized using the same set of controls as above and also the SRHS measure. All of the reported correlations are highly statistically significant with p-values of less than 0.0001.



Table 3 Health Measures, Mortality, and Labor Market Outcomes<sup>1</sup>

	(1) Mortality		(2) Work				(3) Weekly Wages			
	Female	Male	Female		Male		Female		Male	
SRHS										
Fair	0.52**	0.48**	0.13**	0.13**	0.17**	0.13**	36.15	-25	144.56	85.2
	[0.03]	[0.03]	[0.01]	[0.02]	[0.01]	[0.03]	[58.03]	[338.86]	[122.89]	[217.20]
Good	0.29**	0.27**	0.24**	0.23**	0.28**	0.23**	76.43	126.43	214.01+	157.54
	[0.02]	[0.02]	[0.01]	[0.02]	[0.01]	[0.02]	[56.11]	[332.64]	[117.94]	[209.52]
Very Good	0.19**	0.18**	0.29**	0.27**	0.33**	0.26**	175.20**	187.47	402.52**	235.87
	[0.02]	[0.02]	[0.01]	[0.02]	[0.01]	[0.03]	[56.36]	[334.13]	[118.44]	[212.24]
Excellent	0.18**	0.15**	0.31**	0.29**	0.37**	0.32**	231.93**	410.79	461.10**	360.61
	[0.02]	[0.02]	[0.01]	[0.03]	[0.01]	[0.03]	[57.93]	[342.52]	[121.83]	[221.15]
LMI										
1	0.98	0.97	0.09**	0.08**	0.13**	0.13**	72.83+	216.66	95.6	30.94
	[0.07]	[0.08]	[0.01]	[0.02]	[0.01]	[0.03]	[42.97]	[203.02]	[138.04]	[237.23]
2	0.85*	0.80*	0.13**	0.11**	0.19**	0.15**	60.09	21.36	73.26	-42.51
	[0.07]	[0.07]	[0.01]	[0.02]	[0.01]	[0.03]	[40.59]	[193.72]	[129.14]	[223.53]
3	0.83*	0.81*	0.15**	0.10**	0.22**	0.15**	69.49+	84.92	169.05	116.3
	[0.07]	[0.07]	[0.01]	[0.02]	[0.01]	[0.03]	[39.78]	[191.28]	[125.89]	[218.68]
4	0.69**	0.74**	0.17**	0.14**	0.24**	0.20**	124.61**	149.6	325.12**	309.44
	[0.06]	[0.06]	[0.01]	[0.02]	[0.01]	[0.03]	[38.68]	[185.32]	[123.06]	[213.10]
Log Grip Strength				0.03**		0.02*		33.1		89.82
				[0.01]		[0.01]		[66.91]		[60.93]
Exp. Air Flow				0.03**		0.03**		114.28+		87.44*
				[0.01]		[0.01]		[64.18]		[36.57]
Test Statistics (Degrees of Freedom) on Exclusion Tests (Chi-squares for Mortality, F-tests for Work and Wages)										
All Measures	810.15	900.29	900.29	58.26	815.81	52.28	17.68	2.32	14.76	5.25
	(8)**	(8)**	(8,72100)**	(10,7829)**	(8,57457)**	(10,5701)**	(8,25222)**	(10,2639)**	(8,24787)**	(10,2263)**
SRHS	545.92	613.86	614.40	50.50	671.92	39.30	19.57	2.53	12.29	1.73
	(4)**	(4)**	(4,72100)**	(4,7829)**	(4,57457)**	(4,5701)**	(4,25222)**	(4,2639)**	(4,24787)**	(4,2263)
Other Measures <sup>2</sup>	23.56	20.48	196.16	17.17	243.80	18.50	4.74	1.14	8.62	4.58
	(4)**	(4)**	(4,72100)**	(6,7829)**	(4,57457)**	(6,5701)**	(4,25222)**	(6,2639)	(4,24787)**	(6,2263)**
Observations	53,956	42,369	72,138	7,869	57,495	5,741	25,260	2,679	24,825	2,303

<sup>1</sup> Columns 1 and 2 display hazard rates relating mortality up to 2004 to Health Measures estimated with a log-linear correction for the age of individuals. Columns 3-10 report OLS estimates. The dependent variables in column 3-6 are an indicator for working and in column 7-10 the weekly wage conditional on working for pay. Standard errors are reported below the point estimates in brackets. Stars denote significance levels: \*\* p<0.01, \* p<0.05, + p<0.1.

<sup>2</sup> Other Measures refers to the Large Muscle Index, log grip strength, and expiratory air flow when included in the regression.

Table 4 Principal Component Analysis of Health Measures for High School Graduates Age 65-69

Component:	(1)	(2)	(3)	(4)	(5)
Eigenvalue	1.91	0.97	0.83	0.72	0.57
Proportion of Variation	0.38	0.19	0.17	0.14	0.11
Current / Preceding Eigenvalue		0.51	0.91	0.87	0.79
	Eigenvector				
Large Muscle Index	0.46	-0.59	0	0.3	0.6
Self-rated Health Status	0.51	-0.44	0.12	-0.24	-0.7
Peak Expiratory Airflow	0.47	0.35	-0.16	-0.72	0.34
Log Grip Strength	0.42	0.37	-0.63	0.49	-0.21
Log Walking Speed	0.38	0.45	0.75	0.31	0.01

Reported are the eigenvalues and the eigenvectors of the correlation matrix of the health measures for male high school graduates between age 65 and 69. Below the eigenvalues, the share in the overall variation is reported and below this the ratio of the eigenvalue of the current principal component with the preceding one. The principal component analysis is based on 894 respondents with observed values for all five health measures.

Table 5 The Correlations in Health Measures within and across Age (Males)

		Large Muscle Index	Self-rated Health Status	Peak Expiratory Airflow	Log Grip Strength	Log Walking Speed
Age	Large Muscle Index	1				
	Self-rated Health Status	0.44	1			
	Peak Expiratory Airflow	0.18	0.28	1		
	Log Grip Strength	0.19	0.19	0.27	1	
	Log Walking Speed	0.21	0.24	0.27	0.23	1
Age-2	Large Muscle Index	0.62	0.41	0.20	0.16	0.25
	Self-rated Health Status	0.41	0.66	0.27	0.13	0.21
	Peak Expiratory Airflow	0.18	0.30	0.62	0.18	0.45
	Log Grip Strength	0.15	0.18	-0.00	0.74	0.05
	Log Walking Speed	0.15	0.12	0.12	0.08	0.49

Reported are pairwise correlations in residualized health measures within age and between age and age-2. The measures are residualized using height and a full set of age dummies. Note that the correlations across age within clinical measures are often based on few measures due to fact that only subsamples of the HRS waves in more recent years include clinical measures. For example the correlations of the log walking speed variables with the other clinical variables are based only on 19 and 23 observations, respectively.

Table 6: Static Measurement Equation for Selected Groups

Age Group	Male			Female		
	50-54	65-69	80-84	50-54	65-69	80-84
	Parameters of Measurement Equations					
	Factor Loadings					
Self-rated Health Status	7.118 (1.858)	4.422 (0.591)	3.382 (0.585)	3.884 (0.576)	4.859 (0.582)	4.161 (0.665)
Large Muscle Index	5.867 (1.342)	3.992 (0.480)	2.230 (0.315)	3.557 (0.471)	3.449 (0.357)	3.092 (0.407)
Peak Exp. Air Flow	1.749 (0.467)	2.157 (0.263)	1.141 (0.176)	0.953 (0.133)	1.003 (0.112)	0.983 (0.139)
Log Walking Speed	na	1.577 (0.216)	1.744 (0.242)	na	1.856 (0.203)	2.503 (0.393)
	Error Variances					
Log Grip Strength	0.318 (0.069)	0.329 (0.097)	0.413 (0.080)	0.336 (0.050)	0.359 (0.027)	0.451 (0.043)
Peak Exp. Air Flow	0.654 (0.054)	0.779 (0.039)	0.770 (0.045)	0.341 (0.022)	0.307 (0.012)	0.271 (0.014)
Log Walking Speed	na	0.619 (0.051)	0.548 (0.055)	na	0.639 (0.054)	0.724 (0.069)
	Regression Coefficients Height					
SHRS	1.54 (0.275)	1.474 (0.230)	0.922 (0.303)	1.345 (0.219)	0.396 (0.223)	0.307 (0.270)
Large Muscle Index	0.134 (0.249)	-0.138 (0.199)	-0.426 (0.285)	0.154 (0.215)	-0.63 (0.200)	-0.589 (0.249)
Log Grip Strength	2.176 (0.279)	2.332 (0.230)	2.279 (0.378)	1.903 (0.303)	1.557 (0.280)	2.421 (0.385)
Peak Exp. Air Flow	2.885 (0.449)	2.803 (0.378)	2.027 (0.496)	1.949 (0.292)	1.303 (0.238)	0.728 (0.292)
Log Walking Speed	na	0.644 (0.349)	1.040 (0.505)	na	0.549 (0.346)	0.502 (0.583)
	Regression Coefficients Age					
Self-rated Health Status	-0.029 (0.101)	0.161 (0.034)	0.085 (0.043)	0.033 (0.034)	0.115 (0.032)	0.13 (0.037)
Large Muscle Index	-0.010 (0.084)	0.129 (0.031)	0.054 (0.030)	0.050 (0.032)	0.099 (0.023)	0.092 (0.027)
Obs.	6,859	10,590	4,779	11,291	11,930	7,560

The Table shows the estimates of the static measurement model for selected age groups and gender obtained using Mplus. Reported are analytic standard errors.

Table 7: Fit of Static Measurement Model for Females age 50-54

Panel A: Bivariate Frequency Table for Subjective Health Measures		Large Muscle Index (LMI)				
		4	3	2	1	0
SRHS	Poor	3.17 / 2.92	1.56 / 1.54	0.84 / 0.97	0.56 / 0.52	0.45 / 0.39
	Fair	3.30 / 3.13	3.12 / 3.11	2.92 / 2.94	2.61 / 2.61	2.92 / 3.26
	Good	1.95 / 2.04	3.59 / 3.25	5.35 / 4.85	5.87 / 6.09	11.26 / 11.28
	Very Good	0.81 / 0.68	1.93 / 2.09	3.98 / 4.18	6.79 / 6.73	17.61 / 17.63
	Excellent	0.23 / 0.09	0.57 / 0.61	1.37 / 1.80	3.48 / 3.45	13.76 / 13.88

Each cell reports the percentage of individuals who fall into each combination of the SRHS and the LMI. The first number in each cell refers to the observed and the second the predicted frequencies. SRHS refers to the self-reported health status variable. The LMI is constructed from reported difficulties with the following activities: sitting for two hours, getting up from a chair, stooping or kneeling or crouching, pushing or pulling a large object. The number of observations with non-missing values on both the SRHS and LMI among females aged 50-54 is 11,254.

Panel B: Variance Covariances for Objective Health Measures		
	Log Grip	Exp. Air Flow
Log Grip	0.43 / 0.44	
Exp. Air Flow	0.13 / 0.10	0.42 / 0.44

Reported are the observed / estimated covariances of the objective measures available for this age-education group. The number of observations in non-missing values on both log grip and expiratory airflow variables among females aged 50-54 is 842.

Table 8: The Distribution of Health for Selected Age Groups

Age Group		Male			Female		
		50-54	65-69	80-84	50-54	65-69	80-84
		Intercept and Age-regression Coefficient and Variance					
Class 1	Intercept	0.755 (0.765)	3.650 (0.552)	3.897 (1.369)	1.376 (0.567)	1.764 (0.474)	1.712 (0.828)
	Age	0.007 (0.015)	-0.040 (0.008)	-0.049 (0.017)	-0.028 (0.011)	-0.032 (0.007)	-0.032 (0.010)
	Variance	0.028 (0.014)	0.021 (0.007)	0.076 (0.038)	0.106 (0.028)	0.032 (0.007)	0.012 (0.006)
Class 2	Intercept	1.318 (0.755)	3.042 (0.896)	2.096 (1.178)	0.705 (0.419)	0.873 (0.600)	2.230 (0.856)
	Age	0.000 (0.014)	-0.037 (0.012)	-0.021 (0.014)	-0.011 (0.008)	-0.025 (0.009)	-0.043 (0.011)
	Variance	0.006 (0.003)	0.023 (0.030)	0.028 (0.011)	0.002 (0.000)	0.023 (0.019)	0.048 (0.016)
		Logit Parameter and Implied Probability of Class 1					
Logit Parameter		-0.792 (0.343)	1.316 (1.157)	-0.159 (0.623)	0.663 (0.103)	1.292 (0.686)	-0.585 (0.478)
Class 1 Probability		0.31	0.79	0.46	0.66	0.78	0.36
		Implied Means, Std Dev, and Skew in the Distribution of Health Factors					
		1.256	0.882	0.150	-0.011	-0.472	-1.157
Standard Deviation		0.147	0.229	0.335	0.288	0.251	0.269
Skew		-1.121	-0.558	-0.568	-0.608	-0.337	-0.343
Observations		6,859	10,590	4,779	11,291	11,930	7,560

Reported are the estimated parameters pertaining to the health factor distribution from the static measurement model described in the text. Also reported are moments from the implied health factor distribution. Standard errors are in parentheses.

Table 9: Estimates of a Simple Aging Process:  $h_{a+1}=\mu+\rho h_a+\varepsilon$  - No Mortality Correction

Females						
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$
50-54	-0.032	0.995	0.017	No Mortality Model Estimated		-0.032
	[-0.05,-0.02]	[0.89,1.03]	[0.015,0.022]			
55-59	-0.027	0.970	0.012			-0.023
	[-0.05,-0.01]	[0.89,1.03]	[0.004,0.018]			
60-64	-0.032	1.022	0.013			-0.038
	[-0.05,-0.02]	[0.90,1.05]	[0.012,0.021]			
65-69	-0.030	1.020	0.013			-0.039
	[-0.07,-0.02]	[0.94,1.03]	[0.013,0.019]			
70-74	-0.079	0.965	0.020			-0.058
	[-0.11,-0.02]	[0.94,1.04]	[0.009,0.022]			
75-79	-0.024	1.036	0.007	-0.055		
	[-0.12,-0.01]	[0.94,1.05]	[0.006,0.019]			
80-84	-0.024	1.028	0.010	-0.056		
	[-0.11,-0.01]	[0.96,1.04]	[0.010,0.017]			
85-100	-0.116	0.954	0.028	-0.053		
	[-0.12,-0.09]	[0.95,0.97]	[0.028,0.030]			

Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$
50-54	-0.108	1.077	0.016	No Mortality Model Estimated		-0.012
	[-0.19,0.09]	[0.87,1.09]	[0.001,0.019]			
55-59	-0.103	1.069	0.012			-0.021
	[-0.17,0.02]	[1.00,1.12]	[0.001,0.014]			
60-64	0.011	0.955	0.014			-0.035
	[-0.09,0.09]	[0.89,1.04]	[0.011,0.018]			
65-69	-0.057	1.025	0.014			-0.036
	[-0.09,0.03]	[0.93,1.04]	[0.010,0.018]			
70-74	-0.063	1.011	0.017			-0.055
	[-0.10,-0.02]	[0.94,1.08]	[0.015,0.020]			
75-79	-0.033	0.936	0.019	-0.062		
	[-0.07,-0.00]	[0.86,1.02]	[0.017,0.023]			
80-84	-0.017	0.880	0.019	-0.037		
	[-0.05,0.02]	[0.66,0.98]	[0.018,0.030]			
85-100	-0.107	0.799	0.023	-0.091		
	[-0.15,-0.05]	[0.43,0.95]	[0.015,0.037]			

Reported are parameter estimates of the dynamic model obtained using the simulation approach described in the paper. Below the parameter estimates are the 95% confidence intervals obtained by bootstrapping with 100 replications. The change in mean health in the last column is obtained using the parameter estimates presented in the same row.

Table 10: Estimates of a Mortality Corrected Aging Process:  $h_{a+1}=\mu+\rho h_a+\varepsilon$

Females							
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$	Mortality
50-54	-0.031	0.994	0.017	8.167	1.682	-0.031	0.000
	[-0.05,-0.02]	[0.89,1.04]	[0.015,0.022]	[5.1,10.7]	[1.56,2.71]		
55-59	-0.027	0.970	0.012	10.411	2.569	-0.023	0.000
	[-0.05,-0.01]	[0.89,1.04]	[0.004,0.018]	[2.9,10.7]	[0.80,2.51]		
60-64	-0.032	1.021	0.013	7.884	1.676	-0.038	0.000
	[-0.06,-0.02]	[0.90,1.05]	[0.012,0.019]	[3.0,10.6]	[1.33,2.70]		
65-69	-0.030	1.019	0.013	9.924	1.879	-0.039	0.000
	[-0.07,-0.02]	[0.94,1.03]	[0.013,0.019]	[2.7,10.6]	[0.59,2.55]		
70-74	-0.080	0.964	0.023	7.162	1.703	-0.058	0.000
	[-0.11,-0.02]	[0.92,1.04]	[0.011,0.023]	[3.0,10.2]	[1.25,2.61]		
75-79	-0.023	1.037	0.006	7.626	2.152	-0.055	0.000
	[-0.11,-0.01]	[0.95,1.05]	[0.006,0.019]	[3.3,10.0]	[1.27,2.80]		
80-84	-0.024	1.031	0.010	3.751	1.597	-0.060	0.027
	[-0.11,-0.01]	[0.96,1.04]	[0.010,0.018]	[2.5,6.8]	[0.60,2.26]		
85-100	-0.120	0.956	0.028	2.935	1.051	-0.060	0.069
	[-0.19,-0.09]	[0.91,0.97]	[0.022,0.030]	[2.1,3.6]	[0.53,1.44]		

Males							
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$	Mortality
50-54	-0.108	1.077	0.016	8.666	2.425	-0.012	0.000
	[-0.24,0.11]	[0.87,1.14]	[0.003,0.020]	[0.7,8.7]	[0.37,3.46]		
55-59	-0.102	1.069	0.012	0.458	1.801	-0.021	0.005
	[-0.16,-0.02]	[1.00,1.12]	[0.001,0.014]	[-0.2,1.8]	[0.81,2.62]		
60-64	0.006	0.957	0.015	1.228	1.106	-0.038	0.009
	[-0.09,0.08]	[0.89,1.04]	[0.011,0.018]	[0.7,1.9]	[0.61,1.72]		
65-69	-0.058	1.025	0.014	0.648	1.936	-0.037	0.011
	[-0.10,0.03]	[0.93,1.08]	[0.008,0.017]	[0.2,1.2]	[1.32,2.70]		
70-74	-0.065	1.011	0.018	1.087	1.417	-0.057	0.019
	[-0.11,-0.010]	[0.93,1.08]	[0.015,0.020]	[0.7,1.3]	[0.95,1.95]		
75-79	-0.048	0.963	0.019	1.048	1.693	-0.065	0.034
	[-0.08,-0.01]	[0.87,1.04]	[0.015,0.024]	[0.9,1.3]	[1.26,1.99]		
80-84	-0.024	0.894	0.020	1.263	1.648	-0.042	0.062
	[-0.07,0.01]	[0.74,1.02]	[0.015,0.026]	[1.2,1.5]	[1.27,2.25]		
85-100	-0.097	0.799	0.023	1.385	1.702	-0.082	0.104
	[-0.13,-0.05]	[0.52,0.97]	[0.005,0.033]	[1.2,2.2]	[1.48,3.61]		

Presented are parameter estimates of the dynamic model obtained using the simulation approach described in the paper. Below the parameters are 95% confidence intervals obtained by bootstrapping with 100 replications. The change in mean health in the last column is obtained using the parameter estimates presented in the same row. This is the change in health in the population including descendants (i.e. without removing descendants from the population on which the decline in health is calculated).



Table 11: The Random Effects Model

<b>Females</b>							
Standard Deviations of Random Effects							
Expiratory Air Flow	Log Grip Strength	SRHS	LMI				
0.41	0.38	0.71	0.71				
[0.34,0.44]	[0.24,0.41]	[0.70,0.76]	[0.70,0.73]				
Autoregressive Parameters							
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$	Mortality
50-54	-0.029 [-0.04,-0.02]	0.929 [0.85,1.00]	0.062 [0.060,0.064]	8.2 [3.3,10.8]	1.7 [1.5,2.8]	-0.028	0.000
55-59	-0.030 [-0.05,-0.01]	0.949 [0.86,1.01]	0.028 [0.022,0.030]	10.4 [2.9,10.7]	2.6 [0.9,2.6]	-0.022	0.000
60-64	-0.034 [-0.06,-0.03]	0.999 [0.92,1.02]	0.038 [0.037,0.043]	8.1 [3.1,10.6]	1.7 [1.5,2.7]	-0.034	0.000
65-69	-0.041 [-0.08,-0.03]	0.984 [0.90,1.00]	0.021 [0.021,0.028]	9.9 [2.7,10.7]	1.9 [0.6,2.6]	-0.034	0.000
70-74	-0.093 [-0.13,-0.05]	0.938 [0.89,0.99]	0.013 [0.003,0.020]	7.2 [2.9,10.3]	1.7 [1.2,2.7]	-0.055	0.000
75-79	-0.046 [-0.14,-0.02]	1.003 [0.90,1.02]	0.007 [0.011,0.026]	7.7 [3.2,10.1]	2.2 [1.4,2.8]	-0.049	0.000
80-84	-0.099 [-0.20,-0.08]	0.964 [0.88,0.97]	0.090 [0.090,0.093]	3.8 [3.2,6.7]	1.6 [1.2,2.3]	-0.058	0.027
85-100	-0.205 [-0.40,-0.19]	0.893 [0.75,0.91]	0.147 [0.146,0.148]	2.9 [2.2,3.7]	1.1 [0.6,1.5]	-0.058	0.072

<b>Males</b>							
Standard Deviations of Random Effects							
Expiratory Air Flow	Log Grip Strength	SRHS	LMI				
0.61	0.33	0.76	0.78				
[0.55,0.83]	[0.26,0.40]	[0.75,0.80]	[0.74,0.79]				
Autoregressive Parameters							
Age	$\mu$	$\rho$	$\sigma$	$\alpha_0$	$\alpha_1$	$E[h_{a+1}]-E[h_a]$	Mortality
50-54	-0.069 [-0.10,0.28]	1.047 [0.76,1.08]	0.027 [0.014,0.032]	8.8 [0.3,8.8]	2.4 [0.4,3.6]	-0.010	0.000
55-59	-0.082 [-0.15,0.04]	1.054 [0.95,1.11]	0.021 [0.016,0.025]	0.5 [-0.2,1.7]	1.8 [0.8,2.7]	-0.019	0.005
60-64	-0.023 [-0.06,0.11]	0.944 [0.86,1.02]	0.023 [0.015,0.029]	1.2 [0.6,1.7]	1.1 [0.6,1.7]	-0.035	0.008
65-69	-0.032 [-0.07,0.07]	0.996 [0.88,1.04]	0.032 [0.030,0.038]	0.6 [0.2,1.1]	2.0 [1.3,2.7]	-0.036	0.011
70-74	-0.015 [-0.06,0.03]	0.946 [0.87,1.01]	0.012 [0.011,0.025]	1.1 [0.7,1.3]	1.4 [1.0,1.9]	-0.053	0.018
75-79	0.000 [-0.04,0.04]	0.862 [0.76,0.98]	0.099 [0.098,1.01]	1.0 [0.9,1.3]	1.8 [1.3,2.1]	-0.063	0.031
80-84	-0.006 [-0.04,0.03]	0.825 [0.67,0.90]	0.162 [0.161,0.163]	1.3 [1.2,1.5]	1.8 [1.3,2.3]	-0.036	0.051
85-100	-0.101 [-0.13,-0.06]	0.781 [0.50,0.88]	0.251 [0.250,0.252]	1.4 [1.3,2.0]	1.9 [1.6,3.3]	-0.084	0.093

Presented are parameter estimates of the dynamic model obtained using the simulation approach described in the paper. Below the parameters are 95% confidence intervals obtained by bootstrapping with 100 replications. The change in mean health in the last column is obtained using the parameter estimates presented in the same row.

**Table 12: Observed and Predicted Autocorrelation Matrices for Females aged 65-69**

<b>Observed Empirical Moments</b>					
		Period t+2			
Period t		Air Flow	Grip Str.	SRHS	LMI
	Air Flow	0.75	0.20	0.18	0.10
	Grip Str.	0.13	0.41	0.17	0.11
	SRHS	0.23	0.20	0.69	0.43
	LMI	0.11	0.19	0.43	0.65

<b>Predicted Moments</b>					
Basic Specification					
		Period t+2			
Period t		Air Flow	Grip Str.	SRHS	LMI
	Air Flow	0.17	0.16	0.30	0.23
	Grip Str.	0.15	0.16	0.27	0.22
	SRHS	0.30	0.26	0.54	0.44
	LMI	0.24	0.22	0.45	0.38

Mortality Corrected Model					
		Period t+2			
Period t		Air Flow	Grip Str.	SRHS	LMI
	Air Flow	0.17	0.16	0.30	0.23
	Grip Str.	0.15	0.16	0.26	0.21
	SRHS	0.30	0.26	0.54	0.44
	LMI	0.24	0.22	0.45	0.37

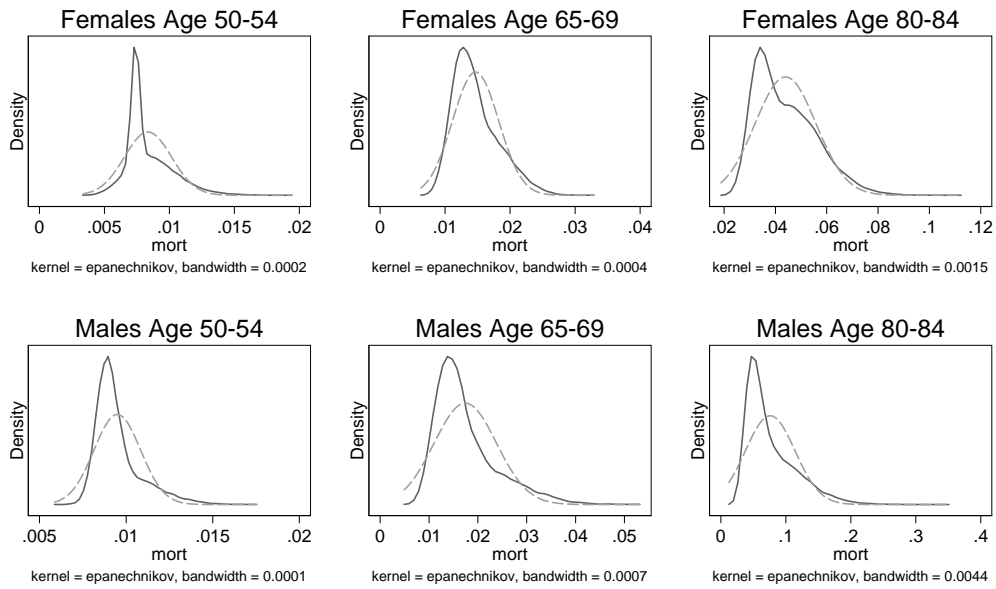
Measurement Specific Random Effect Model					
		Period t+2			
Period t		Air Flow	Grip Str.	SRHS	LMI
	Air Flow	0.61	0.15	0.29	0.23
	Grip Str.	0.15	0.67	0.27	0.21
	SRHS	0.29	0.26	0.58	0.43
	LMI	0.23	0.21	0.44	0.53

Correlations of health measures across HRS-interviews for females aged 65-69. The top panel displays the empirically observed moments, whereas the bottom three panels show the correlations implied by the estimated dynamic models.

Table 13: The Distribution of Health under Different Normalizations

Implied Means, Std Dev, and Skew in the Distribution of Health Factors						
Age Group	Male			Female		
	50-54	65-69	80-84	50-54	65-69	80-84
Log Grip Strength						
Mean	1.26	0.88	0.15	-0.01	-0.47	-1.16
Standard Deviation	0.15	0.23	0.34	0.29	0.25	0.27
Skew	-1.12	-0.56	-0.57	-0.61	-0.34	-0.34
Predicted Mortality (percent)						
Mean	0.95	1.79	7.60	0.83	1.47	4.39
Standard Deviation	0.13	0.69	3.81	0.18	0.35	1.19
Skew	1.50	1.30	1.30	1.25	0.82	0.78
Relative Mortality Risk (to Mean)						
Mean	1.00	1.00	1.00	1.00	1.00	1.00
Standard Deviation	0.14	0.36	0.25	0.22	0.24	0.27
Skew	1.50	1.30	1.30	1.25	0.82	0.78

Figure 3: Estimated Health Densities  
In Units of Predicted Mortality



Also shown are normal densities with same mean and variance.